PC12 Sequences and Series Canvas

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- Common ratio, first term, general term
- Geometric sequences connecting to exponential functions
- Infinite geometric series
- Sigma notation

1. $7, 10, 13, \ldots$ State the value of the first term t_1.

2. $7, 10, 13, \ldots$ What is the value of the common difference d?

3. $7, 10, 13, \ldots$ What is the value of the 5^{th} term?

4. $7, 10, 13, \ldots$ State the value of the 100^{th} term.

5. $5, 3\frac{2}{3}, \frac{7}{3}, \ldots$ Find t_{1000}.

6. There are initially 50 ants in a colony. Each month 80 ants are added. Approximately how many months would it take for the ant population to become an established colony of 3000 ants?

7. Which of the following sequences is arithmetic?
   $15, 12, 9, \ldots$
   $-20, -15, -10, \ldots$
   $1, 1, 2, 3, 5, 8, 11, \ldots$
   $1, 2, 3, 4, \ldots$
   $4, 8, 16, \ldots$

8. Given $t_n = 5n + 3$. State the value of the fourth term.

9. Given $t_n = 2 - 4n$. Which term corresponds to a value of -38?

10. True or False: The arithmetic series formula
    $S_n = \frac{n}{2}(t_1 + t_n)$ is equivalent to $\frac{n}{2}(2t_1 + (n-1)d)$

11. When proving that $S_n = \frac{n}{2}(2t_1 + (n-1)d)$
    Proof of $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ formula:
    $S_n = t_1 + (t_1 + d) + (t_1 + 2d) + (t_1 + 3d) + (t_1 + 4d) + \cdots + (t_1 + (n-2)d) + (t_1 + (n-1)d)$
    Writing this series backwards:
    $S_n = (t_1 + (n-1)d) + (t_1 + (n-2)d) + \cdots + (t_1 + 4d) + (t_1 + 3d) + (t_1 + 2d) + (t_1 + d)$
    Now add these two equations:
    $2S_n = [2t_1 + (n-1)d] + [2t_1 + (n-1)d] + [2t_1 + (n-1)d] + \cdots$
    These terms in square brackets repeat n times:
    $2S_n = n[2t_1 + (n-1)d]$

    What is the next step of this proof?

12. The value of the second term is -3. Given the common difference is 4, state the value of the fifth term.

13. Find the sum of all the two digit even numbers between 9 and 99.

14. Find the sum of all the three digit odd numbers between 100 and 200.

15. The value of the third term is 4 and the value of the 10^{th} term is 25.
Find the sum of the first three numbers of this arithmetic series.

16. In an arithmetic series $S_2 = 19$ and $S_4 = 50$. Find S_{20}.

17. True or False: $t_n = t_1 r^n$ is the geometric sequence formula.

18. $3, 6, 12, \ldots$ Find the value of the 6^{th} term.

19. $3, 6, 12, \ldots$ Find the sum of the first 10 terms of this series.

20. Given the first term is 2 and value of the 6^{th} term
is 486, find the value or the common ratio r.

21. Given the third term is 16 and the 9th term is 1024, find the value of the first term.

22. In a geometric sequence, the third term is 16 and the 7th term is 256.
Find the possible value(s) of r.

23. $5 + 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + \cdots + 5\left(\frac{2}{3}\right)^{100}$
Find the value of r (common ratio).

24. $5 + 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + \cdots + 5\left(\frac{2}{3}\right)^{100}$
How many terms are in this series?

25. Evaluate $5 + 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + \cdots + 5\left(\frac{2}{3}\right)^{100}$

26. True or False: $S_n = \frac{t_1(1-r^n)}{1-r} = \frac{t_1(r^n-1)}{r-1}$

27. True or False: $\frac{r^n-1}{r-1} = r^{n-1} + r^{n-2} + \cdots + r^3 + r^2 + r + 1$

28. The sum of this series $5, 10, 20, 40 \ldots$ is 5115.
Find the number of terms in this series.

29. Given $S = 4 + 0.2 + 0.01 + 0.005 + \cdots$
Evaluate S_{10} accurate to TWO decimal points.

30. In a geometric sequence, the second term is 28, and the 5th term is 1792.
Find S_8 the sum of the first 8 terms.

31. Find the value of x in the following geometric sequence:
$3, 12, 48, 5x + 7$

32. Given the first term is 8 and the common ratio is $\frac{1}{3}$. Find the value of S_∞.

33. When does S_∞ converge?

34. Given the first term is 10 and the common ratio is 2. Find the value of S_∞.

35. Does the following series converge or diverge? $-20, -\frac{200}{9}, -\frac{2000}{81}, \ldots$

36. Why does the formula $\frac{t_1(1-r^n)}{1-r}$ becomes $\frac{a}{1-r}$?

37. $t_1 = 8. t_3 = 2. t_0 = 16. t_5 = \frac{1}{2}. t_6 = \frac{1}{4}$. Find S_∞.

38. $2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$ Find S_∞.

39. $t_n = f(n) = 2(3)^n$. Evaluate $f(3)$.

40. $f(1) = 0.2$
$f(2) = 0.2 + 0.02$
$f(3) = 0.2 + 0.02 + 0.002$
Evaluate $f(n)$ in the form a/b
given $n \to \infty$ (x approaches infinity) i.e. $\lim_{n\to\infty} f(n)$

41. You initially have \$100. Each year your investment grows by 10%.
How much money will you have by the end of year 10?
Round to the nearest dollar.

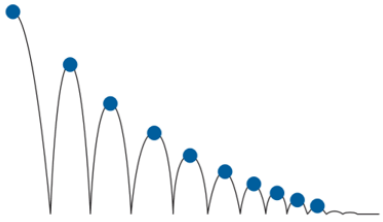
42. You invest \$500 at the start of each year. If the annual investment return is 8% how much money will you have in total at the end of 10 years? Round to the nearest dollar.

43. The sum of $4 + 12 + 36 + 108 + \cdots + t_n$ is 4372.
How many terms are in the series?

44. The sum of the infinite geometric sequence is 25.
The common ratio is $\frac{4}{5}$. Find the first term.

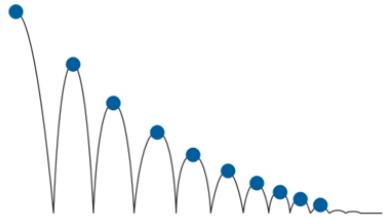
45. If $0.\overline{23}$ is written as infinite geometric series,
find r as a decimal number.

46. See diagram below:



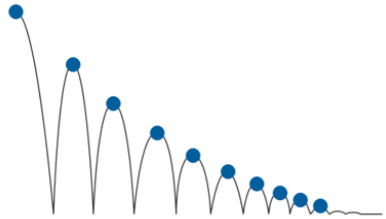
A ball is dropped from a height of 12 m.
After each bounce it rises to 80% of its previous height.
What height does the ball reach after the 4th bounce?
Round to ONE decimal place.

47. See diagram below:



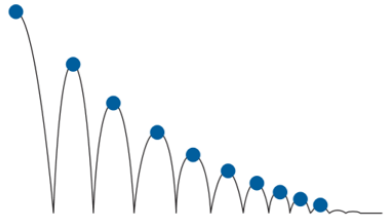
A ball is dropped from a height of 12 m.
After each bounce it rises to 80% of its previous height.
After how many bounces will the ball reach an approximate height of 10 cm?

48. See diagram below:



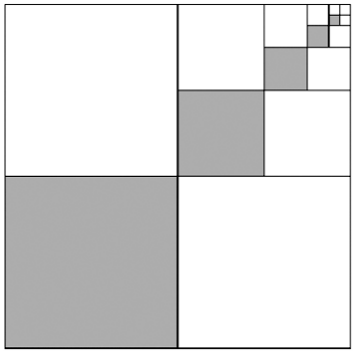
A ball is dropped from a height of 12 m.
After each bounce it rises to 80% of its previous height.
What is the total vertical distance travelled by the ball as soon
as the ball bounces 10 times? Round to ONE decimal place.

49. See diagram below:



A ball is dropped from a height of 12 m.
After each bounce it rises to 80% of its previous height.
What is the total distance travelled by the ball if it bounces indefinitely?

50. See diagram below:



The largest square (that contains every other square) measures 8 by 8.
The pattern goes on towards infinity.
Find the total area of the shaded region in the form a/b.

51. Evaluate $\sum_{2}^{5}(3x)$

52. Evaluate $\sum_{-1}^{3}(1-4x)$

53. Evaluate $\sum_{0}^{2}(x^3-x)$

54. $20+17+14+11+8$ is equivalent to $y=ax+b$.

55. $20+17+14+11+8$ is equivalent to $\sum_{1}^{a}(bx+c)$.

56. Model $2(3)^4 + 2(3)^5 + 2(3)^6 + 2(3)^7$ using sigma notation.

57. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} = \sum_1^5 \frac{r+1}{?}$

What should replace the ? part?

58. $\sum_0^{10} 2(3)^n$. How many terms exist?

59. Evaluate $\sum_0^{10} 2(3)^n$

60. Consider an arithmetic sequence $a_1, a_2, a_3, \ldots$
with first term $a_1 = 3$ and common difference $d = 2$.

A second sequence b_n is constructed such that each term is defined as $b_n = \left(\frac{1}{2}\right)^{a_n}$.
Is b_n and arithmetic or geometric sequence?

61. Consider an arithmetic sequence $a_1, a_2, a_3, \ldots$
with first term $a_1 = 3$ and common difference $d = 2$.

A second sequence b_n is constructed such that each term is defined as $b_n = \left(\frac{1}{2}\right)^{a_n}$.

Evaluate $S = \sum_{n=1}^{\infty} b_n$ in the form a/b.